Redshift of last scattering

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Free electron fraction:

$$x = \frac{N_e}{N_{H^0} + N_p} = \frac{N_e}{N_H} \qquad N_e = x N_H = x \alpha N_B \quad (\alpha = 0.75)$$

Thomson cross section:

$$\sigma_T = \frac{8\pi}{3}\left(\frac{e^2}{4\pi\varepsilon_0 m_e c^2}\right)^2 \simeq 6.65 \cdot 10^{-29}\ m^2$$

Probability of Thomson scattering in $[t, t+dt]$:

$$dP = \sigma_T N_e c\, dt$$

Average number of scatterings in $[t, t_0]$:

$$N = \int_t^{t_0} dP = \int_t^{t_0} \sigma_T N_e c\, dt' = \sigma_T c \int_t^{t_0} N_e(t')\, dt' = \mathcal{T}(t, t_0)$$

Thomson optical depth

Probability of ZERO scatterings between t and t_0:

$$\bar{P}(t, t_0) = e^{-\mathcal{T}(t, t_0)}$$

$\Rightarrow$ Probability of scattering for the last time in $[t, t+dt]$:

$$g(t)\, dt = e^{-\mathcal{T}(t, t_0)} \sigma_T N_e c\, dt$$

$g(t)$: Thomson visibility function

$$\rightarrow g(z) = g(t)\left|\frac{dt}{dz}\right| = g(t)\frac{1}{(1+z)H(z)}$$

$$g(z) = \frac{1}{(1+z)\, H(z)} \sigma_T c\, N_e(z)\, e^{-\mathcal{T}(z, \emptyset)} = \frac{\sigma_T c \alpha \eta n_\gamma}{(1+z)\, H(z)} x(z)\, e^{-\mathcal{T}(z, \emptyset)}$$

$$\mathcal{T}(z, \emptyset) = \sigma_T c \int_\emptyset^z \alpha \eta n_\gamma x(z') \frac{1}{(1+z')H(z')} dz'$$

$$\boxed{g(z) = y(z)\, e^{-\int_\emptyset^z y(z')dz'}} \qquad y(z) = \sigma_T c \alpha \eta n_\gamma \frac{x(z)}{(1+z)\, H(z)}$$