First acoustic peak in the CMB

Peter Raffai
peter.raffai@ttk.elte.hu

Physical size of the sound horizon at decoupling:

$$r_s = \int_{\emptyset}^{t_*} \frac{a_*}{a} c_s \, dt = \frac{1}{1+z_*} \int_{\emptyset}^{t_*} \frac{c_s}{a} dt \qquad dt = -\frac{1}{H_0} \frac{dz}{(1+z)E(z)} \qquad a = \frac{1}{1+z}$$

Speed of sound in photon-baryon plasma:

$$c_s^2 = \frac{dp}{d\varrho} = \frac{c^2}{3(1+R)} \quad \text{where} \quad R = \frac{\varrho_b + \frac{p_b}{c^2}}{\varrho_r + \frac{p_r}{c^2}} = \frac{3}{4}\frac{\varrho_b}{\varrho_r} = \frac{3}{4}\frac{\Omega_{b,0}}{\Omega_{\gamma,0}}(1+z)^{-1}$$

$$\Rightarrow r_s = \frac{1}{1+z_*} \frac{1}{\sqrt{3}} \frac{c}{H_0} \int_{z_*}^{+\infty} \frac{dz}{E(z)\sqrt{1+R(z)}}$$

$$\hookrightarrow \boxed{\theta_* = \frac{r_s}{d_A(z_*)}} \qquad \Rightarrow d_A(z_*) = \frac{d_M(z_*)}{1+z_*} \qquad \Rightarrow d_M(z_*) = \begin{cases} \frac{1}{\sqrt{\Omega_{k,0}}} \frac{c}{H_0} \sinh\left(\sqrt{\Omega_{k,0}} \frac{H_0}{c} d(z_*)\right) & \text{if } \Omega_{k,0} > \emptyset \\ d(z_*) & \text{if } \Omega_{k,0} = \emptyset \\ \frac{1}{\sqrt{|\Omega_{k,0}|}} \frac{c}{H_0} \sin\left(\sqrt{|\Omega_{k,0}|} \frac{H_0}{c} d(z_*)\right) & \text{if } \Omega_{k,0} < \emptyset \end{cases}$$

$$\Downarrow$$

$$d(z_*) = \frac{c}{H_0} \int_{\emptyset}^{z_*} \frac{dz}{E(z)}$$

$$E(z) = \sqrt{\Omega_{m,0}(1+z)^3 + \Omega_{r,0}(1+z)^4 + \Omega_{\Lambda,0} + \Omega_{k,0}(1+z)^2}$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{r,0} - \Omega_{\Lambda,0}$$

$$\Omega_{r,0} = \frac{\varrho_{r,0}}{\varrho_{c,0}} = \frac{8\pi G}{3H_0^2} \frac{g_* \pi^2}{30 \hbar^3 c} \left(\frac{k_B T_0}{c}\right)^4$$

$T_0 = 2.72548 \pm 0.00057$ K (Fixsen, D. J., ApJ 707, 2, 916-920, 2009)

$g_* = 2$

$$\Omega_{r,0} = \Omega_{\gamma,0} \cdot \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{\frac{4}{3}} N_\nu\right] \qquad = \left(\frac{T_\nu}{T_0}\right)^4$$

$N_\nu \simeq 3$ ($N_{eff} = 3.046$)

↓ number of neutrino species

↘ correction for non-thermal distortion of ν-spectra due to $e^- + e^+$ annihilation after ν-decoupling