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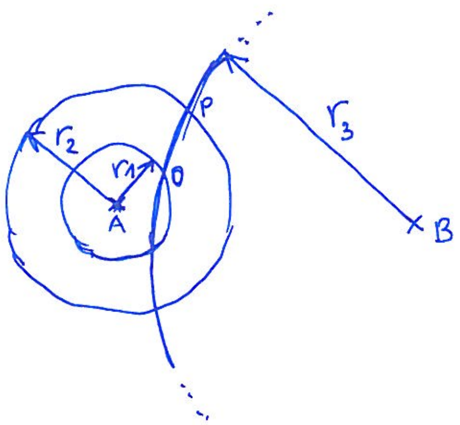
(1) Copernican principle: we are not „special" or „privileged" observers in the universe

$\Rightarrow$ (above a certain size scale) the universe looks the same all around (i.e. it is isotropic) regardless of where the observer is



$\rightarrow \rho_A(r_1) = \text{const.} \; (= \rho_0)$

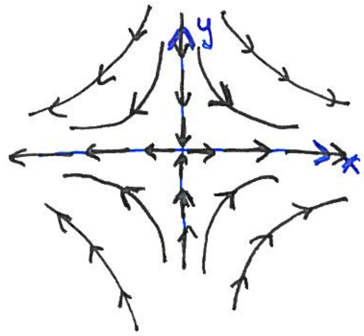
$\rightarrow \rho_B(r_3) = \text{const.} \; (= \rho_0 = \rho_P)$

$\rightarrow \rho_A(r_2) = \text{const.} \; (= \rho_P)$

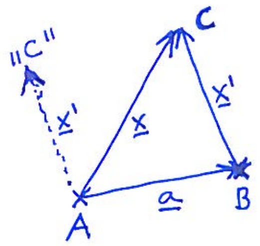
$\rho_A(r_1) = \rho_B(r_3) = \rho_A(r_2) = \underline{\underline{\rho}}$

(2) An example for a homogeneous but NOT isotropic universe (/velocity field):

$$\underline{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$$



(3) Velocity field:



$\rightarrow \quad \underline{x} = \underline{x}' + \underline{a} \quad \Rightarrow \quad \underline{x}' = \underline{x} - \underline{a} \quad (1)$

$\rightarrow \quad \underline{v}(\underline{x}, t) = \underline{v}'(\underline{x}', t) + \underline{v}(\underline{a}, t)$

A: $\underline{v}(\underline{\emptyset}, t) = \underline{\emptyset}$

$\underline{v}'(\underline{x}', t) = \underline{v}(\underline{x}, t) - \underline{v}(\underline{a}, t) \quad / \leftarrow (1)$

B: $\underline{v}'(\underline{\emptyset}, t) = \underline{\emptyset}$

$\underline{v}'(\underline{x} - \underline{a}, t) = \underline{v}(\underline{x}, t) - \underline{v}(\underline{a}, t)$

$\rightarrow \quad \underline{v}(\underline{x}', t) \overset{!}{=} \underline{v}'(\underline{x}', t)$

$\Rightarrow \quad \underline{v}(\underline{x} - \underline{a}, t) = \underline{v}(\underline{x}, t) - \underline{v}(\underline{a}, t)$

(1) $\underline{x} = \underline{\emptyset} \rightarrow \underline{v}(-\underline{a}, t) = -\underline{v}(\underline{a}, t)$

(2) $\underline{a} = -\underline{b} \rightarrow \underline{v}(\underline{x} - (-\underline{b}), t) = \underline{v}(\underline{x}, t) - \underline{v}(-\underline{b}, t)$

$\underline{v}(\underline{x} + \underline{b}, t) = \underline{v}(\underline{x}, t) + \underline{v}(\underline{b}, t)$

(3) $\underline{v}(N\underline{x} + M\underline{b}, t) = N\underline{v}(\underline{x}, t) + M\underline{v}(\underline{b}, t)$

Linear relationship between $\underline{x}$ and $\underline{v}$!

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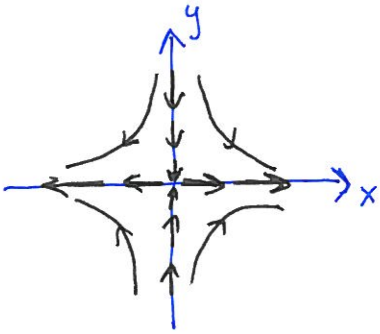
$$\boxed{\underline{v}(\underline{x},t) = \underline{\underline{C}}\,\underline{x}}$$

$\underline{\underline{C}} \rightarrow$ linear operator ($\underline{\underline{C}} = \underline{\underline{C}}(t)$)

$$v_i = C_{ik} x_k$$

In 2D:

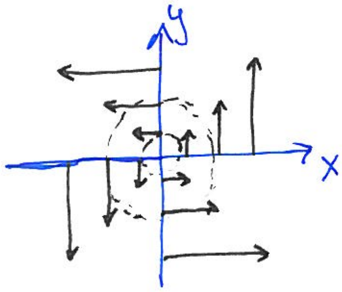
(a) $C_{ik} = \begin{pmatrix} 2 & \emptyset \\ \emptyset & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 2 & \emptyset \\ \emptyset & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ -y \end{pmatrix}$



$\Rightarrow$ homogeneous? YES!
isotropic? NO!

(b) $C_{ik} = \begin{pmatrix} \emptyset & -1 \\ 1 & \emptyset \end{pmatrix} \rightarrow$ anti-symmetric tensor

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \emptyset & -1 \\ 1 & \emptyset \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$$



$\Rightarrow$ homogeneous?
YES!
isotropic?
YES! (in 2D!)

$$\Rightarrow C_{ik}(t) = S_{ik}(t) + A_{ik}(t) = \underbrace{\frac{1}{2}(C_{ik} + C_{ki})}_{S_{ik}} + \underbrace{\frac{1}{2}(C_{ik} - C_{ki})}_{\cancel{A_{ik}}}$$

$\cancel{A_{ik}} \Rightarrow$ does not satisfy isotropy (above 2D)

$\Rightarrow$ All symmetric matrices can be diagonalized!

$$S_{ik} = \begin{pmatrix} S_{11} & & \emptyset \\ & S_{22} & \\ \emptyset & & S_{33} \end{pmatrix}$$

But due to isotropy: $S_{11} \overset{!}{=} S_{22} \overset{!}{=} S_{33} = H(t)$

$$\Rightarrow \underline{\underline{S}}(t) = H(t)\,\underline{\underline{I}} \quad \Rightarrow \quad v_i(\underline{x},t) = S_{ik}(t)\,x_k = H(t)\,\delta_{ik}\,x_k = H(t)\,x_i$$

$$\boxed{\underline{v}(\underline{r},t) = H(t)\,\underline{r}}$$

Hubble – Lemaître law

$H(t)$: Hubble parameter

$H(\text{today}) = H_0 \doteq$ Hubble constant