Notes on dark energy

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1.

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) = -\frac{4\pi G}{3}(1+3w)\rho$$

Dark energy:

$$1 + 3w_{DE} < \emptyset$$

$$\boxed{w_{DE} < -\frac{1}{3}}$$

Cosmological constant:

$$w_\Lambda = -1 \left(< -\frac{1}{3}\right)$$

2.

$$\rho(a) = \rho_0 a^{-3(1+w)} = \rho_0 a^n$$

$$w = -\frac{1}{3} \Rightarrow n = -2$$

$$w_{DE} < -\frac{1}{3} \Rightarrow n > -2$$

$$w_\Lambda = -1 \Rightarrow n = \emptyset$$

$$w < -1 \Rightarrow n > \emptyset$$

$$w > -1 \Rightarrow n < \emptyset$$

$n > \emptyset \Rightarrow$ DE condensates with expansion $\Rightarrow$ „Big Rip"

$n < \emptyset \Rightarrow$ DE dilutes with expansion $\Rightarrow$ heat death

3.

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

FI:

$$G_{\emptyset\emptyset} + \Lambda g_{\emptyset\emptyset} = \frac{8\pi G}{c^4} T_{\emptyset\emptyset}$$

$$\frac{3}{c^2}\left(\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right) - \Lambda = \frac{8\pi G}{c^4}\rho c^2$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$1 = \frac{8\pi G}{3H^2}\rho - \frac{kc^2}{a^2H^2} + \frac{\Lambda c^2}{3H^2} = \Omega_{tot} + \Omega_k + \Omega_\Lambda$$

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} \Rightarrow \rho_\Lambda = \frac{\Lambda c^2}{8\pi G} = \text{const.}$$

$$\rho_\Lambda = \rho_{\Lambda,0}\, a^{\emptyset}$$

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FII:

$$g^{\mu\nu}G_{\mu\nu} + \Lambda \underbrace{g^{\mu\nu}g_{\mu\nu}}_{4\Lambda} = \frac{8\pi G}{c^4} g^{\mu\nu}T_{\mu\nu}$$

$$-R + 4\Lambda = \frac{8\pi G}{c^4}\left(-\rho c^2 + 3p\right)$$

$$-\frac{6}{c^2}\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2}\right) + 4\Lambda = -\frac{8\pi G}{c^2}\left(\rho - \frac{3p}{c^2}\right)$$

$$\frac{\ddot{a}}{a} + \frac{8\pi G}{3}\rho - \cancel{\frac{kc^2}{a^2}} + \frac{\Lambda c^2}{3} + \cancel{\frac{kc^2}{a^2}} - \frac{2\Lambda c^2}{3} = \frac{4\pi G}{3}\left(\rho - \frac{3p}{c^2}\right)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2} - \frac{\Lambda c^2}{4\pi G}\right)$$

$$\| $$

$$-2\rho_\Lambda = (1+3w_\Lambda)\rho_\Lambda$$

$$\Downarrow$$

$$1 + 3w_\Lambda = -2$$

$$\boxed{w_\Lambda = -1}$$

4.

$$T^{\mu\nu} = \left(\rho + \frac{p}{c^2}\right)u^\mu u^\nu + p g^{\mu\nu} \qquad u^\mu = (c, 0, 0, 0)$$

$$T^{\mu\nu} = (1+w)\rho u^\mu u^\nu + w\rho c^2 g^{\mu\nu}$$

$w_\Lambda = -1 \Rightarrow T^{\mu\nu} = -\rho c^2 g^{\mu\nu} \Rightarrow$ independent of u^μ!

vacuum energy?

5.

Einstein 1917:

$$0 = \frac{8\pi G}{3}\rho_{m,0} - kc^2 + \frac{\Lambda c^2}{3} \quad \Rightarrow \quad k = \frac{4\pi G \rho_{m,0}}{c^2} > 0$$

$$0 = -\frac{4\pi G}{3}\rho_{m,0} + \frac{\Lambda c^2}{3} \quad \Rightarrow \quad \Lambda = \frac{4\pi G \rho_{m,0}}{c^2} > 0$$