Cosmological distance measures

Peter Raffai
peter.raffai@ttk.elte.hu

(1) Hubble distance (d_H):

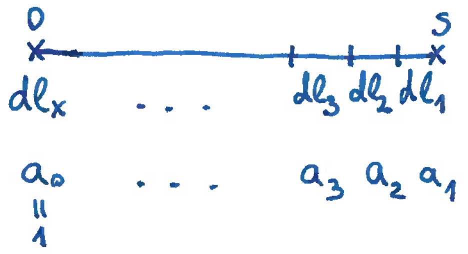
$$d_H = cT_H = \frac{c}{H_0}$$

(2) Light-travel distance OR lookback distance (d_T):

$$d_T(z) = c\,t_{\text{lookback}}(z) = d_H \int_0^z \overbrace{\frac{dz'}{(1+z')\underbrace{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{r,0}(1+z')^4 + \Omega_{\Lambda,0} + \Omega_{k,0}(1+z')^2}}_{E(z';[\Omega_{m,0};\Omega_{r,0};\Omega_{\Lambda,0}])}}}^{\text{"}dl\text{"}}$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{r,0} - \Omega_{\Lambda,0}$$

(3) Proper distance (d):



→ How much did $dl_1, dl_2, dl_3 \ldots dl_x$ expand until photons reached observer „O" from source „S"?

$$\left.\begin{aligned} dl_1 &\to \frac{a_0}{a_1} dl_1 = \frac{1}{a_1} dl_1 \\ dl_2 &\to \frac{a_0}{a_2} dl_2 = \frac{1}{a_2} dl_2 \\ &\vdots \\ dl_x &\to \frac{a_0}{a_0} dl_x = dl_x \end{aligned}\right\}$$

By integrating them, we get the actual distance of the source today!

$$\times \frac{1}{a} = \times (1+z') \Rightarrow \boxed{d(z) = d_H \int_0^z \frac{dz'}{E(z')}}$$

At any other cosmic time „t":

$$t \to a(t) \times \to \frac{1}{1+z(t)} \times$$

$$\boxed{D = \frac{d_H}{1+z(t)} \int_0^z \frac{dz'}{E(z')}} \left(= a(t) d(z)\right)$$

(4) Comoving distance (d_c):

From notes on peculiar velocities: $D = R(t)\chi = a(t) R_0 \chi = a(t) d_c$

$$d = a_0 d_c = d_c$$

$$\boxed{d_c = d_H \int_0^z \frac{dz'}{E(z')}}$$

Proper distance (d or D) changes with time, comoving distance (d_c) does not!

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(5) Transverse comoving distance (d_M):

Def.: Two objects at the same d_c and θ angular distance from each other have a distance of $d_M \cdot \theta$.

$$d_M(z) = \begin{cases} \frac{d_H}{\sqrt{\Omega_{k,0}}} \, sh\left(\sqrt{\Omega_{k,0}} \frac{d_c}{d_H}\right) & \text{if} \quad \Omega_{k,0} > \emptyset \Leftrightarrow k < \emptyset \\ d_c & \text{if} \quad \Omega_{k,0} = \emptyset \Leftrightarrow k = \emptyset \\ \frac{d_H}{\sqrt{|\Omega_{k,0}|}} \sin\left(\sqrt{|\Omega_{k,0}|} \frac{d_c}{d_H}\right) & \text{if} \quad \Omega_{k,0} < \emptyset \Leftrightarrow k > \emptyset \end{cases}$$

(6) Angular diameter distance (d_A)

r: Physical size of a source object (at the time of emission)

$$\frac{r}{d_A} = \frac{d_M \theta}{d_M} = \theta \quad \Rightarrow \quad d_A = a(t_e) d_M \quad \Rightarrow \quad \boxed{d_A(z) = \frac{d_M}{1+z}}$$

$$d_A = \frac{r}{\theta}$$

(7) Luminosity distance (d_L)

$$f = \frac{L'}{4\pi d_M^2}$$

L: $\frac{hc}{\lambda}$ energy photons are emitted, N number within dt duration.

$$L = \frac{N}{dt} \cdot \frac{hc}{\lambda} \Rightarrow L' = \frac{N}{dt'} \cdot \frac{hc}{\lambda'} \qquad dt' = (1+z)dt$$

$$\lambda' = (1+z)\lambda$$

$$L' = \frac{1}{(1+z)^2} \frac{N}{dt} \cdot \frac{hc}{\lambda} = \frac{L}{(1+z)^2}$$

$$f = \frac{L}{4\pi [(1+z) d_M]^2} = \frac{L}{4\pi d_L^2(z)} \Rightarrow \boxed{d_L(z) = (1+z) d_M}$$

Exercise

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Particle horizon (r_p) = comoving distance of the horizon

Let us emit a light signal at $t=t_1$ when $a(t_1)=a_1$.
What is the comoving distance of the signal at $t=t_2$ when $a(t_2)=a_2$?

→ At t_1 the comoving distance is $d_c(t_1)=0$.

→ Along the radial trajectory of the light signal:

$$ds = d\theta = d\phi = 0$$

$$c\,dt = R(t)\,d\chi$$

$$c\,dt = a(t)\,R_0\,d\chi \qquad d_c = R_0\chi$$

$$d(d_c) = c\,\frac{dt}{a} \qquad d(d_c) = R_0\,d\chi$$

$$\Rightarrow \quad r_p = c\int_0^{t_2} \frac{dt}{a(t)}$$

$$d(d_c) = c\,\frac{dt}{da}\,\frac{da}{a} = c\,\frac{da}{\dot{a}\,a} = c\,\frac{da}{Ha^2} = \frac{c}{H_0}\,\frac{da}{a^2E(a)}$$

$$d_c(a_1,a_2) = \frac{c}{H_0}\int_{a_1}^{a_2}\frac{da}{a^2E(a)}$$

$$E(a) = \sqrt{\Omega_{m,0}a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}a^{-2}}$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{r,0} - \Omega_{\Lambda,0}$$

→ Observable today: $a_1 = 0 \qquad a_2 = 1$

→ Max. reachable: $a_1 = 1 \qquad a_2 = +\infty$

→ Max. observable: $a_1 = 0 \qquad a_2 = +\infty$