About the origin of CMB anisotropies

Peter Raffai
peter.raffai@ttk.elte.hu

(1) $$Y = \frac{m_{He}}{m} = \frac{m_{He}}{m_H + m_{He}} = \frac{4 m_p N_{He}}{m_p N_H + 4 m_p N_{He}} = \frac{4 N_{He}}{N_H + 4 N_{He}}$$

$$Y N_H + 4 Y N_{He} = 4 N_{He}$$

$$N_{He} = \frac{Y}{4(1-Y)} N_H$$

$$N = N_H + N_{He} = N_H + \frac{Y}{4(1-Y)} N_H = \frac{4-3Y}{4(1-Y)} N_H = \frac{4-3Y}{Y} N_{He}$$

$$\rho = \frac{\alpha m_p N}{V} = \frac{m_H + m_{He}}{V} = \frac{m_p N_H + 4 m_p N_{He}}{V} = \frac{m_p N}{V} \underbrace{\left[\frac{4(1-Y)}{4-3Y} + 4\frac{Y}{4-3Y}\right]}_{\alpha = \frac{1}{1-\frac{3}{4}Y}}$$

$$Y \simeq 0.25 \Rightarrow \alpha \simeq 1.23$$

(2) $$\left.\begin{array}{l} T \sim a^{-1} \\ \rho \sim a^{-3} \end{array}\right\} \quad \rho \sim T^3$$

$$\delta\rho \sim 3T^2 \delta T$$

$$\frac{\delta\rho}{\rho} = 3\frac{\delta T}{T} \qquad \left\langle \frac{\delta\rho}{\rho} \right\rangle = 3\left\langle \frac{\delta T}{T} \right\rangle$$

(3) $$\left\langle \frac{\delta\rho}{\rho} \right\rangle = \left\langle \frac{\delta N}{N} \right\rangle = \frac{1}{\sqrt{N}} = \left(\frac{\rho V}{\alpha m_p}\right)^{-\frac{1}{2}} = \left[\frac{\rho_{m,0}(1+z_*)^3}{\alpha m_p} \frac{4\pi r_*^3}{3}\right]^{-\frac{1}{2}} = \left[\frac{\Omega_{m,0}(1+z_*)^3}{\alpha m_p} \cdot \frac{3H_0^2}{8\pi G} \cdot \frac{4\pi r_*^3}{3}\right]^{-\frac{1}{2}}$$

$$\left\langle \frac{\delta\rho}{\rho} \right\rangle = \frac{1}{H_0}\sqrt{\frac{2\alpha G m_p}{\Omega_{m,0}(1+z_*)^3 r_*^3}} = 3\left\langle \frac{\delta T}{T} \right\rangle$$

$$\left\langle \frac{\delta T}{T} \right\rangle = \frac{1}{3H_0}\sqrt{\frac{2\alpha G m_p}{\Omega_{m,0}(1+z_*)^3 r_*^3}}$$

$\alpha \simeq 1.23$

$G = 6.67 \cdot 10^{-11} \frac{Nm^2}{kg^2}$

$m_p = 1.67 \cdot 10^{-27}$ kg

$$\left.\begin{array}{l} H_0 = 67.66 \frac{km}{s \cdot Mpc} \\ z_* = 1089.8 \\ r_* = 144.57 \text{ Mpc} \end{array}\right\} \text{Planck 2018}$$

$$\Rightarrow \boxed{\left\langle \frac{\delta T}{T} \right\rangle \simeq 3.7 \cdot 10^{-43} \ll 10^{-5}}$$