Constraints of inflation

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$z_* = 1089.80 \pm 0.21$

$r_* = 144.57 \pm 0.22$ Mpc

Planck 2018 (A&A 641, A6, 67, 2020)

$\Omega_{m,0} = 0.3111 \pm 0.0056$

$\Omega_{r,0} = 9.10 \cdot 10^{-5}$ (Fixsen 2009)

$\Omega_{\Lambda,0} = 0.6889 \pm 0.0056$

$H_0 = 67.66 \pm 0.42 \ \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$

$r_{Pl} = 1.6 \cdot 10^{-35}\,\text{m} \simeq 10^{-35}\,\text{m}$

$$a_* = \frac{1}{1+z_*} = 9 \cdot 10^{-4} \simeq 10^{-3}$$

Beginning and end of inflation: $t_b \simeq 10^{-36}$ s $\quad t_e \simeq 10^{-32}$ s

Scale factor at the end of inflation:

$$t_e = \frac{1}{H_0}\int_{\emptyset}^{1} \frac{da}{a E(a)} - \frac{1}{H_0}\int_{a_e}^{1} \frac{da}{a E(a)} \qquad E(a) = \sqrt{\Omega_{m,0}a^{-3} + \Omega_{r,0}a^{-4} + \Omega_{\Lambda,0}} \quad (\Omega_{k,0} \overset{!}{=} \emptyset)$$

Approximation: $t_e = \left(\frac{a_e}{a_*}\right)^2 t_*$ (assuming radiation domination)

$\Rightarrow t_e \simeq 10^{-32}\,\text{s} \Rightarrow a_e \simeq 10^{-26}$

Inflation: $a_e = a_b e^N = a_b e^{H_i \Delta t}$

$H_i = \frac{N}{\Delta t} \quad (\Delta t \simeq 10^{-32}\,\text{s})$

$$\hookrightarrow \quad r_* = \frac{a_*}{a_e} r_e = \frac{a_*}{a_e} e^N r_b$$

$$r_b = \frac{a_e}{a_*} r_* e^{-N} \overset{!}{\lesssim} r_{Pl} \quad (\text{for } N = N_{max})$$

$$N_{max} = \ln\left(\frac{a_e}{a_*}\frac{r_*}{r_{Pl}}\right) \simeq \underline{\underline{85}}$$

To solve horizon problem: $N_{min} \simeq 60$ (since $\frac{1}{a_e} = 10^{26} \simeq e^{60}$)

To solve the flatness problem: $\Omega_{k,0} = 0.0007 \pm \underbrace{0.0019}_{\sigma_k}$ (Planck 2018)

$\sigma_k \simeq 10^{-3}$

$$|\Omega_{k,0}| \overset{!}{<} \sigma_k$$

$$\frac{H_0^2}{H_e^2}|\Omega_{k,0}| a_e^{-2} < \frac{H_0^2}{H_e^2}\sigma_k a_e^{-2} \qquad H_e = H_i = \frac{N}{\Delta t} = H_b = \text{const.}$$

$$|\Omega_{k,e}| < \frac{H_0^2}{H_e^2}\sigma_k a_e^{-2} \qquad H_0 \simeq \frac{1}{T} \quad (T = \text{age of the universe})$$

$$|\Omega_{k,b}| = \frac{H_e^2}{H_b^2}|\Omega_{k,e}|\left(\frac{a_b}{a_e}\right)^{-2} < \cancel{\frac{H_e^2}{H_b^2}}\frac{H_0^2}{H_e^2}\sigma_k a_e^{-2}\left(\frac{a_b}{a_e}\right)^{-2} = \frac{1}{N^2}\left(\frac{\Delta t}{T}\right)^2 e^{2N_{min}} e^{2N}\sigma_k$$

$\nearrow |\Omega_{k,b}| < 10^{20}$ if $N = 85$

$\searrow |\Omega_{k,b}| < 2.5 \cdot 10^{-2}$ if $N = 60$