Matter power spectrum

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(1) Matter-radiation equality

$$\left.\begin{aligned}\Omega_m &= \frac{H_0^2}{H^2}\Omega_{m,0}\,a^{-3}\\ \Omega_r &= \frac{H_0^2}{H^2}\Omega_{r,0}\,a^{-4}\end{aligned}\right\}\quad \Omega_m(a_{eq}) \stackrel{!}{=} \Omega_r(a_{eq})$$

$$\frac{H_0^{\cancel{2}}}{\cancel{H^2}}\Omega_{m,0}\,a_{eq}^{-3} = \frac{H_0^{\cancel{2}}}{\cancel{H^2}}\Omega_{r,0}\,a_{eq}^{-4}$$

$$a_{eq} = \frac{\Omega_{r,0}}{\Omega_{m,0}} = \frac{9.10\cdot 10^{-5}}{0.3111} \simeq 2.9\cdot 10^{-4}$$

$$z_{eq} = \frac{1}{a_{eq}} - 1 \simeq 3418 \Rightarrow t_{eq} \simeq 51\text{ kyr};\ T_{eq} \simeq 9318\text{ K}$$

(Recombination: $z_* \simeq 1090 \Rightarrow t_* \simeq 370$ kyr; $T_* \simeq 2974$ K)

(2) Jeans wavelength

$$\lambda_J = \sqrt{\frac{\pi}{\rho_m G}}\,c_s = 2\pi\sqrt{\frac{2}{3\Omega_m}}\,\frac{c_s}{c}\left(\frac{c}{H}\right)$$

For $z > z_*$: $\quad c_s \simeq \frac{c}{\sqrt{3}} \Rightarrow \lambda_J = \frac{2\pi}{3}\sqrt{\frac{2}{\Omega_m}}\,\frac{c}{H} \gtrsim \frac{c}{H}\ (\simeq ct)$

For $z < z_*$: $\quad c_s \simeq \emptyset \Rightarrow \lambda_J \simeq \emptyset$

(3) Growth of δ for baryonic matter

	Newtonian approximation	Relativistic calculation
Radiation dominated era (RDE, $z > z_{eq}$)	$\lambda_J \gg \lambda \Rightarrow \delta \simeq$ const. $\lambda_J \ll \lambda \Rightarrow \delta =$ const.	$ct \gg \lambda \Rightarrow \delta \simeq$ const. $ct \ll \lambda \Rightarrow \delta \sim a^2 \longrightarrow ?$
Matter dominated era (MDE, $z \lesssim z_{eq}$)	$\lambda_J \ll \lambda \Rightarrow \delta \sim a$ ($\lambda_J \simeq \emptyset$)	$ct \gg \lambda \Rightarrow \delta \sim a$ $ct \ll \lambda \Rightarrow \delta \sim a \longrightarrow ?$

See: Padmanabhan, T., „Structure formation in the universe", CUP (1995)

? ⇒ Consider a spherical region of radius λ ($> ct$) containing matter with a mean density ρ_1, embedded in a $k = \emptyset$ universe of density ρ_0 (with $\rho_1 = \rho_0 + \delta\rho$, $\delta\rho \ll \rho_0$ and $\delta\rho > \emptyset$). Due to spherical symmetry, the inner region is not affected by the matter outside, hence it

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evolves independently as a $k>\emptyset$ universe.

Thus:

$$H_1^2 = \frac{8\pi G}{3}\rho_1 - \frac{kc^2}{a_1^2} \qquad H_1 = \frac{\dot{a}_1}{a_1}$$

$$H_0^2 = \frac{8\pi G}{3}\rho_0 \qquad H_0 = \frac{\dot{a}_0}{a_0}$$

Let $H_1 = H_0$ at time t:

$$\frac{8\pi G}{3}\rho_0 = \frac{8\pi G}{3}\rho_1 - \frac{kc^2}{a_1^2}$$

$$\rho_1 - \rho_0 = \frac{3}{8\pi G}\frac{kc^2}{a_1^2} \Rightarrow \delta = \frac{\delta\rho}{\rho_0} = \frac{3}{8\pi G}\frac{kc^2}{\rho_0 a_1^2}$$

If $\delta \ll 1 \Rightarrow a_1 \simeq a_0 = a$

$$\delta \sim \frac{1}{\rho_0 a^2} \begin{cases} \text{RDE: } \rho_0 = \rho_{r,0}\, a^{-4} \Rightarrow \delta \sim a^2 \\ \text{MDE: } \rho_0 = \rho_{m,0}\, a^{-3} \Rightarrow \delta \sim a \end{cases}$$

(4) Entering λ_J / the horizon

$$\lambda_J \stackrel{!}{=} \lambda \quad \text{at } a = a_{in}$$

$$\frac{2\pi a_{in}}{\lambda_J} = \frac{2\pi a_{in}}{\lambda} = k \quad \text{(comoving wavenumber)}$$

$$k = \frac{2\pi a_{in}}{\lambda_J} = \frac{2\pi a_{in}}{2\pi}3\sqrt{\frac{\Omega_m}{2}}\frac{H}{c} = 3\sqrt{\frac{\Omega_m}{2}}\frac{a_{in}H}{c} \simeq \frac{a_{in}H}{c}$$

$$\left(\Rightarrow ct \stackrel{!}{=} \lambda \Rightarrow \text{RDE: } \frac{c}{2H} = \lambda \Rightarrow k = 4\pi\frac{a_{in}H}{c} \simeq \frac{a_{in}H}{c}\right)$$

In RDE: $\Omega_m \simeq \emptyset \quad \Omega_r \simeq 1$

$$H = H_0\sqrt{a^{-4}} = H_0 a^{-2}$$

$$k = \frac{a_{in}}{c}H_0 a_{in}^{-2} = \frac{H_0}{c}a_{in}^{-1} = \frac{H_0}{c}(1+z_{in})$$

For k_{eq}:

$$H = H_0\sqrt{\Omega_{m,0}a_{eq}^{-3} + \Omega_{r,0}a_{eq}^{-4}} = H_0\sqrt{2\Omega_{m,0}}\,a_{eq}^{-\frac{3}{2}}$$

$$k_{eq} = \frac{a_{eq}}{c}H_0\sqrt{2\Omega_{m,0}}\,a_{eq}^{-\frac{3}{2}} = \frac{H_0}{c}\sqrt{2\Omega_{m,0}}\,a_{eq}^{-\frac{1}{2}} = \frac{H_0}{c}\sqrt{2\Omega_{m,0}}\,(1+z_{eq})^{\frac{1}{2}}$$

(5) Power spectrum

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For $z_{in} > z_{eq}$ $(k > k_{eq})$: $\delta(k,z) = \delta_0(k) \frac{(1+z_0)^2}{(1+z_{in})^2} \frac{(1+z_{eq})}{(1+z)} = \frac{H_0^2}{c^2} \delta_0(k) \frac{(1+z_0)^2}{k^2} \frac{(1+z_{eq})}{(1+z)}$

For $z_{in} < z_{eq}$ $(k < k_{eq})$: $\delta(k,z) = \delta_0(k) \frac{(1+z_0)^2}{(1+z_{eq})^2} \frac{(1+z_{eq})}{(1+z)}$

$$P(k,z) = \frac{1}{(2\pi)^3} |\delta(k,z)|^2 = P_0(k)(1+z_0)^4 \frac{(1+z_{eq})^2}{(1+z)^2} \cdot \begin{cases} \frac{H_0^4}{c^4} k^{-4} & \text{for } k > k_{eq} \\ (1+z_{eq})^{-4} & \text{for } k < k_{eq} \end{cases}$$

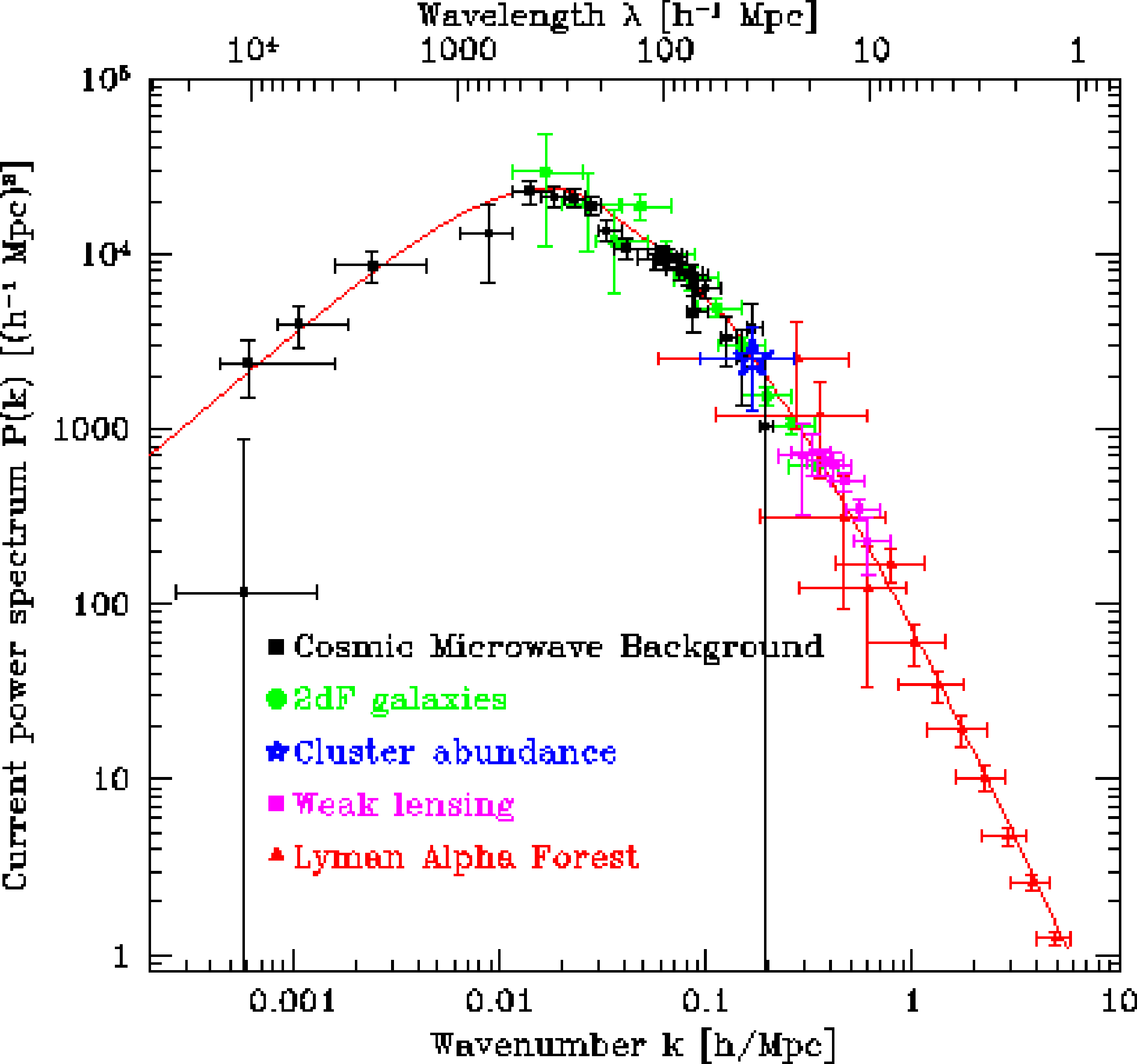
Harrison-Zel'dovich spectrum: $P_0(k) = A_0 k^{n_s}$ with $n_s \simeq 1$

$$\boxed{P(k,z) = A_0 (1+z_0)^4 \frac{(1+z_{eq})^2}{(1+z)^2} \cdot \begin{cases} \frac{H_0^4}{c^4} k^{-4+n_s} & \sim k^{-3} & \text{for } k > k_{eq} \\ (1+z_{eq})^{-4} k^{n_s} & \sim k^{1} & \text{for } k < k_{eq} \end{cases}}$$

$$\Rightarrow \quad k_{eq} \simeq 1.5385 \cdot 10^{-2} \frac{h}{\mathrm{Mpc}} = 1.5385 \cdot 10^{-2} \frac{H_0 \frac{\mathrm{s} \cdot \cancel{\mathrm{Mpc}}}{\mathrm{km}}}{100\ \cancel{\mathrm{Mpc}}} = 1.5385 \cdot 10^{-4}\ H_0 \frac{\mathrm{s}}{\mathrm{km}}$$

$$1 + z_{eq} \simeq \left[1.5385 \cdot 10^{-4}\ H_0 \frac{\mathrm{s}}{\mathrm{km}} \cdot \frac{c}{H_0} \frac{1}{\sqrt{2\Omega_{m,0}}} \right]^2 \simeq 3419$$

$$z_{eq} \simeq 3418$$


Wavelength λ [h⁻¹ Mpc]
10⁴
1000
100
10
1
10⁵
10⁴
1000
100
10
1
Current power spectrum P(k) [(h⁻¹ Mpc)³]
Cosmic Microwave Background
2dF galaxies
Cluster abundance
Weak lensing
Lyman Alpha Forest
0.001
0.01
0.1
1
10
Wavenumber k [h/Mpc]