Matter-dominated universe with $k \neq 0$:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi \cancel{G}}{\cancel{3}} \rho_{m,0}\, a^{-3} \cdot \frac{\cancel{3} H_0^2}{\cancel{8\pi G}} \cdot \frac{1}{\rho_{c,0}} - \frac{kc^2}{a^2}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \Omega_{m,0}\, a^{-3} - \frac{kc^2}{a^2} \quad \xrightarrow{t=t_0} \quad H_0^2 = H_0^2 \Omega_{m,0} - kc^2$$

Let us introduce: $d\eta = \frac{dt}{a}$ conformal time: $\eta$

$$\frac{\dot{a}}{a} = \frac{da}{dt}\frac{1}{a} = \frac{da}{d\eta}\cdot\frac{d\eta}{dt}\cdot\frac{1}{a} = \frac{1}{a^2}\frac{da}{d\eta}$$

$$\frac{1}{a^4}\left(\frac{da}{d\eta}\right)^2 = H_0^2 \Omega_{m,0}\, a^{-3} - \frac{kc^2}{a^2}$$

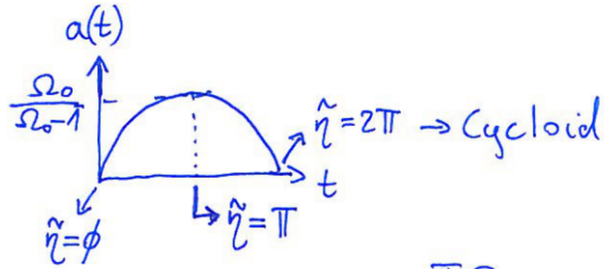
$$\left(\frac{da}{d\eta}\right)^2 = H_0^2 \Omega_{m,0}\, a + H_0^2 (1 - \Omega_{m,0}) a^2$$

$\tilde{\eta} = H_0 \eta \sqrt{\Omega_0 - 1}$ if $\Omega_0 > 1$

$\tilde{\eta} = H_0 \eta \sqrt{1 - \Omega_0}$ if $\Omega_0 < 1$

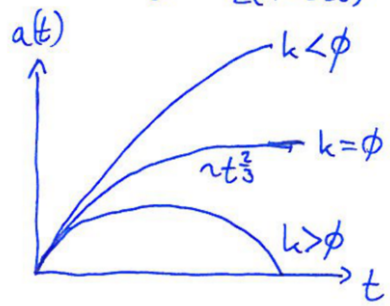
$$\pm\left(\frac{da}{d\tilde{\eta}}\right)^2 = \frac{\Omega_0}{\Omega_0 - 1} a - a^2$$

For $k > 0$: $a(\tilde{\eta}) = \frac{\Omega_0}{2(\Omega_0 - 1)}(1 - \cos\tilde{\eta})$




$$t(\tilde{\eta}) = \frac{\Omega_0}{2H_0(\Omega_0 - 1)^{3/2}}(\tilde{\eta} - \sin\tilde{\eta}) \quad \rightarrow \quad t_{max} = t(2\pi) = \frac{\pi\Omega_0}{H_0(\Omega_0 - 1)^{3/2}}$$

For $k < 0$: $a(\tilde{\eta}) = \frac{\Omega_0}{2(1 - \Omega_0)}(\mathrm{ch}\,\tilde{\eta} - 1)$ $\quad t(\tilde{\eta}) = \frac{\Omega_0}{2H_0(1 - \Omega_0)^{3/2}}(\mathrm{sh}\,\tilde{\eta} - \tilde{\eta})$



$$\frac{H^2}{H_0^2} = \Omega_{m,0}\, a^{-3} + \Omega_{r,0}\, a^{-4} + \Omega_{\Lambda,0} + \Omega_{k,0}\, a^{-2}$$

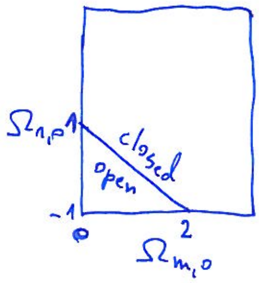
Let us ignore $\Omega_{r,0} \approx \phi$!

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{\Lambda,0}$$

$$\frac{H^2}{H_0^2} = \Omega_{m,0}(a^{-3} - a^{-2}) + \Omega_{\Lambda,0}(1 - a^{-2}) + a^{-2}$$

1. $\Omega_0 = 1 \quad \Rightarrow \quad \Omega_{\Lambda,0} + \Omega_{m,0} = 1$

$$\Omega_{\Lambda,0} = 1 - \Omega_{m,0}$$




2. Let us assume: $\Omega_{m,0} = \phi$

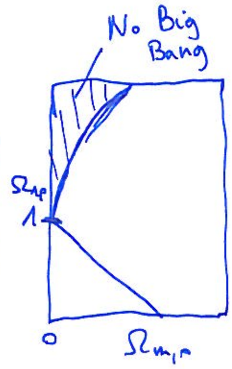
$$\frac{H^2}{H_0^2} = \Omega_{\Lambda,0}(1 - a^{-2}) + a^{-2}$$

Is there a $\phi < a_x < 1$, where $\frac{H^2}{H_0^2}$ becomes $= \phi$?

$$\Omega_{\Lambda,0}(1 - a_x^{-2}) + a_x^{-2} = \phi$$

$$a_x^2 = 1 - \frac{1}{\Omega_{\Lambda,0}} \quad \longrightarrow a_x < 1 \text{ if } \Omega_{\Lambda,0} > \phi$$

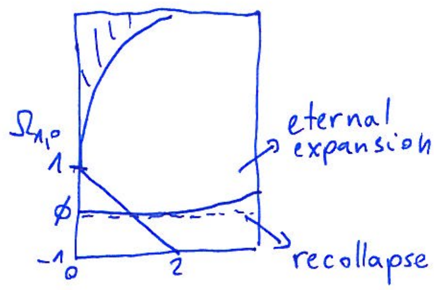
$$\searrow a_x > \phi \text{ if } \Omega_{\Lambda,0} > 1$$




3. Let us assume: $\Omega_{m,0} = \phi$

Is there a $a_x > 1$, where $\frac{H^2}{H_0^2}$ becomes $= \phi$?

$$a_x^2 = 1 - \frac{1}{\Omega_{\Lambda,0}} \overset{!}{>} 1 \Rightarrow \Omega_{\Lambda,0} < \phi$$




4. Around $a(t) = a(t_0) = a_0 (= 1)$

$$a(t) \approx a_0 + \dot{a}\big|_{t=t_0}(t - t_0) + \frac{1}{2}\ddot{a}\big|_{t=t_0}(t - t_0)^2 + \ldots \quad \text{Taylor-series expansion}$$

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2$$

$$q_0 = -\frac{\ddot{a}_0 a_0}{\dot{a}_0^2} \longrightarrow q = -\frac{\ddot{a}\, a}{\dot{a}^2} = -\frac{\ddot{a}}{a H^2} \quad \text{deceleration parameter}$$

$q_0 = \emptyset \quad \text{if} \quad \ddot{a}_0 = \emptyset$

$$\dot{a}^2 = H_0^2 \left( \Omega_{m,0}\, a^{-1} + \Omega_{\Lambda,0}\, a^2 + \Omega_{k,0} \right)$$

$$\dot{a} = \pm H_0 \sqrt{\ldots}$$

$$\ddot{a} = \pm H_0 \frac{1}{2} \frac{1}{\sqrt{\ldots}} \cdot \left( 2a\dot{a}\,\Omega_{\Lambda,0} - a^{-2}\dot{a}\,\Omega_{m,0} \right)$$

$$-\frac{\ddot{a}a}{\dot{a}^2} = \mp H_0 \frac{1}{2} \frac{1}{\sqrt{\ldots}} \left( 2\frac{a^2}{\dot{a}}\Omega_{\Lambda,0} - \frac{a^{-1}}{\dot{a}}\Omega_{m,0} \right)$$

$$q_0 = -\frac{\ddot{a}_0 a_0}{\dot{a}_0^2} = \mp H_0 \frac{1}{2} \frac{1}{\sqrt{\ldots}} \frac{1}{\dot{a}_0} \left( 2\Omega_{\Lambda,0} - \Omega_{m,0} \right)$$

$q_0 = \emptyset \quad \text{if:} \quad 2\Omega_{\Lambda,0} - \Omega_{m,0} = \emptyset$

$$\Omega_{\Lambda,0} = \frac{1}{2}\Omega_{m,0}$$