Peculiar velocities

Peter Raffai
peter.raffai@ttk.elte.hu

FLRW metric:

$$ds^2 = -d(ct)^2 + R^2(t)\left[d\chi^2 + S_k^2(\chi)\,d\theta^2 + S_k^2(\chi)\sin^2\theta\, d\phi^2\right]$$

where: $R(t) = a(t) R_0$

	$k=0$	$k>0$	$k<0$
R_0	R_0	$\frac{1}{\sqrt{k}}$	$\frac{1}{\sqrt{\lvert k\rvert}}$
$S_k(\chi)$	χ	$\sin\chi$	$\mathrm{sh}\,\chi$
$\tilde{r}$	χ	$\sin\chi$	$\mathrm{sh}\,\chi$

(1) Along $d(ct) = d\theta = d\phi = 0$

$$ds^2 = R^2(t)\, d\chi^2$$

$$ds = R(t)\, d\chi \qquad / \int$$

$$\int_0^D ds = R(t) \int_0^\chi d\chi'$$

$$D = R(t)\chi \qquad / \frac{d}{dt} \qquad \text{D: proper distance}$$

$$v_r = \dot{R}\chi + R\dot{\chi} \qquad v_r\text{: recessional velocity}$$

$$v_r = \frac{\dot{R}}{R} R\chi + R\dot{\chi}$$

$$v_r = HD + v_{pec} \qquad v_{pec} = R\dot{\chi} \text{ (peculiar velocity)}$$

(2) Along a radial trajectory of a light beam ($ds = d\theta = d\phi = 0$)

$$0 = -d(ct)^2 + R^2(t)\, d\chi^2$$

$$c^2 dt^2 = R^2 d\chi^2$$

$$c\, dt = R\, d\chi$$

$$c = R(t)\frac{d\chi}{dt}$$

$$\boxed{c = R\dot{\chi} \ (= v_{pec})}$$

Peter Raffai
peter.raffai@ttk.elte.hu

(3) Along a radial trajectory of a body propagating at $v \neq c > 0$ $(d\theta = d\phi = 0)$

$$ds^2 = -d(ct)^2 + R^2(t)\,d\chi^2$$

$$-c^2 d\tau^2 = -c^2 dt^2 + R^2 d\chi^2$$

$$c^2 dt^2\left(1 - \left(\frac{d\tau}{dt}\right)^2\right) = R^2 d\chi^2$$

$$c^2 dt^2\left(1 - \left(1 - \frac{v^2}{c^2}\right)\right) = R^2 d\chi^2$$

$$\cancel{c^2} dt^2 \frac{v^2}{\cancel{c^2}} = R^2 d\chi^2$$

$$v = R(t)\frac{d\chi}{dt}$$

$$\boxed{v = R\dot{\chi}\,(= v_{pec})}$$

$\Rightarrow$ $v_{pec} = v_{pec}(t) = ?$

A particle with v_{pec} in the local comoving frame travels a distance $v_{pec} \cdot dt$ within dt, and enters a new frame with velocity $H(v_{pec} \cdot dt)$. The peculiar velocity of the particle will therefore be reduced by $dv_{pec} = -H \cdot v_{pec} \cdot dt$ when measured in the new comoving frame. Thus:

$$\dot{v}_{pec} = -H v_{pec} \qquad \Rightarrow \text{„Hubble drag"}$$

$$\frac{\dot{v}_{pec}}{v_{pec}} = -\frac{\dot{a}}{a}$$

$$(\ln v_{pec})^{\dot{}} = (\ln a^{-1})^{\dot{}}$$

$$\boxed{v_{pec} = v_{pec,0}\, a^{-1}}$$

$\rightarrow$ de Broglie wavelength: $\lambda = \frac{h}{p} = \frac{h}{m v_{pec}} = \frac{h}{m v_{pec,0}\, a^{-1}} = \frac{h}{p_0} a = \lambda_0 a$

$\rightarrow$ $v_{pec} \sim a^{-1} \Rightarrow E_{kin} \sim v_{pec}^2 \sim a^{-2} \Rightarrow T_{matter} \sim a^{-2}$

$\rightarrow$ $a \rightarrow 0 \Rightarrow v_{pec} \rightarrow +\infty$ $(\rightarrow c)$ $\quad \left(T_{radiation} \sim \frac{hc}{\langle\lambda\rangle} \sim a^{-1}\right)$

BGV theorem: universes that (on average) expand are past-incomplete.

[Borde, Guth, Vilenkin; PRL 90, 15, 151301 (2003)]