Photon propagation

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1. In a homogeneous universe

Geodesic equation:

$$\frac{d^2x^\alpha}{d\lambda^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0$$

$$p^\alpha = \frac{dx^\alpha}{d\lambda} = \left(\frac{E}{c}, p^i\right)$$

$\alpha = 0$:

$$\frac{d}{d\lambda} = \frac{dx^0}{d\lambda}\frac{d}{dx^0} = \frac{E}{c}\frac{d}{d(ct)} = \frac{E}{c^2}\frac{d}{dt}$$

$' = \frac{d}{d(ct)}$ $\quad \dot{} = \frac{d}{dt}$

$$\Gamma^0_{00} = \Gamma^0_{0i} = \Gamma^0_{i0} = 0$$

$$\Gamma^0_{ij} = \frac{a'}{a}g_{ij}$$

$$\Rightarrow \frac{E}{c^2}\frac{d}{dt}\left(\frac{E}{c}\right) + \frac{a'}{a}g_{ij}p^ip^j = 0$$

$$\frac{E}{c^2}\dot{E} + \frac{\dot{a}}{a}g_{ij}p^ip^j = 0$$

$$\hookrightarrow g_{ij}p^ip^j = -a\frac{E}{c^2}\frac{dE}{da}$$

For photons: $ds^2 = 0 \Rightarrow g_{\mu\nu}p^\mu p^\nu = 0$

$$-\frac{E}{c^2} + g_{ij}p^ip^j = 0$$

$$-\frac{E}{c^2} - a\frac{E}{c^2}\frac{dE}{da} = 0$$

$$\frac{dE}{E} = -\frac{da}{a}$$

$$d(\ln E) = d(\ln a^{-1})$$

$$E = \frac{E_0}{a} \qquad E = h\nu = \frac{hc}{\lambda} = \frac{hc}{\lambda_0 a} \Rightarrow \lambda = \lambda_0 a$$

$$\lambda \sim a$$