Relationship between recessional velocity and redshift in an expanding universe

Peter Raffai
peter.raffai@ttk.elte.hu

$$v = H_0 d$$

$$v = H_0 \cdot \frac{c}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_{m,0}(1+z')^3 + \Omega_{r,0}(1+z')^4 + \Omega_{\Lambda,0} + \Omega_{k,0}(1+z')^2}}$$

$$\Omega_{k,0} = 1 - \Omega_{m,0} - \Omega_{r,0} - \Omega_{\Lambda,0}$$

$$\frac{v}{c} \simeq \int_0^z \left(1 + \left[\Omega_{\Lambda,0} - \frac{\Omega_{m,0}}{2} - \Omega_{r,0} - 1\right] z'\right) dz' = z + \frac{1}{4}\left(2\Omega_{\Lambda,0} - \Omega_{m,0} - 2\Omega_{r,0} - 2\right) z^2$$

$$\boxed{\frac{v}{c} \simeq z}$$

Relative error:

$$e = \frac{1}{4}\left|2\Omega_{\Lambda,0} - \Omega_{m,0} - 2\Omega_{r,0} - 2\right| z \overset{!}{<} e_{max}$$

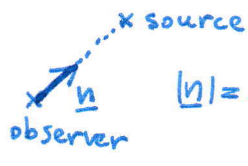
$$z < \frac{4 e_{max}}{\left|2\Omega_{\Lambda,0} - \Omega_{m,0} - 2\Omega_{r,0} - 2\right|} \simeq 4.285 e_{max}$$

For example: $e_{max} = 0.01$ (1%) $\Rightarrow$ $z < 0.043$

$e_{max} = 0.1$ (10%) $\Rightarrow$ $z < 0.43$

Also:

$$1 + z_{obs} = (1 + z_{MW})(1 + z_p)(1 + z_H)$$



$|\underline{n}| = 1$

$z_H = z$ (see above)

$$z_p \simeq \frac{\underline{v}_p \underline{n}}{c}$$

$\underline{v}_p = \underline{v}_g + \underline{v}_s$ (peculiar velocities)

$\underline{v}_g$: source's host galaxy; $\underline{v}_s$: source

$$z_{MW} \simeq \frac{\underline{v}_{MW} \underline{n}}{c}$$

$\underline{v}_{MW}$: Milky Way's peculiar velocity relative to the fundamental frame